Accuracy and resolution of ultrasonic distance measurement with high-time-resolution cross-correlation function obtained by single-bit signal processing

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Abstract: Distance measurement using an ultrasonic wave is suitable for environment recognition in autonomous mobile robots. Ultrasonic distance measurement with the pulse-echo method is based on the determination of the reflected echo’s time of flight (TOF). Pulse compression can improve distance resolution and the reflected echo’s signal-to-noise ratio (SNR). However, calculation of cross correlation requires high-cost digital signal processing. A sensor signal processing method of cross correlation using a delta-sigma modulated single-bit digital signal has been proposed. Cross correlation by single-bit signal processing reduces the calculation costs of cross correlation. Furthermore, cross correlation by single-bit signal processing improves the time resolution of the cross-correlation function. Therefore, the high-time-resolution cross-correlation function improves the distance resolution of the cross-correlation method. In this paper, ultrasonic distance measurement using cross correlation by single-bit signal processing is evaluated based on computer simulations and the experimental results.

Keywords: Ultrasonic distance measurement, Cross correlation, Single-bit signal processing, Delta-sigma modulation

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1. INTRODUCTION

Autonomous mobile robots require many kinds and large numbers of sensors to measure the distance, velocity, and scale of objects for environment recognition. The signal processing of each sensor requires real-time environment recognition with high accuracy and resolution, despite the limited computational ability in the signal processing for each sensor. Distance measurements using ultrasonic waves are widely used in industrial applications because structures easily reflect them. Additional advantages of ultrasonic sensors include their low purchase cost, small size, and simple hardware. Therefore, ultrasonic distance measurements have been studied and used for environment recognition in autonomous mobile robots [1–3].

The method of ultrasonic distance measurement is based on the pulse-echo method, which determines an ultrasonic wave’s time of flight (TOF) [4]. The TOF is the interval from the transmission of an ultrasonic pulse to the reception of an echo reflected from an object. The object’s distance is estimated from the product of the TOF and the propagation velocity of an ultrasonic wave. To effectively improve the distance resolution and the signal-to-noise ratio (SNR) of the reflected echo, the pulse compression technique has been employed in the pulse-echo method [5]. A frequency-modulated (FM) signal or signal coding by pseudo-random sequences is typically transmitted in the cross-correlation method, which is the pulse-echo method with pulse compression [2,3,5–7]. In the case of a linear-frequency-modulated (LFM) signal, a received signal is correlated with a reference LFM signal. The TOF of the received LFM signal is estimated from the maximum peak time in the cross-correlation function of the received signal and the reference LFM signal.

Cross-correlation operation consists of huge iterations of multiplications and accumulations, and requires high-
cost digital signal processing. Therefore, real-time distance measurements by the cross-correlation method are difficult because of the limited computational ability of autonomous mobile robots. To reduce the calculation costs of cross correlation, a sensor signal processing method using a delta-sigma modulated single-bit digital signal has been proposed [8–10]. Cross correlation by single-bit signal processing in the proposed method consists of a recursive cross-correlation operation of single-bit signals and a smoothing operation accomplished by a moving average filter [10]. The calculation costs of cross correlation are reduced by the recursive cross-correlation operation of single-bit signals.

In the cross-correlation method, the time resolution of the received LFM signal’s TOF is typically improved by interpolating the cross-correlation function with its phase or improving the sampling frequency of digital signal processing. The former requires a large lookup table of arctangent and complex cross-correlation operation, whereas the latter greatly increases the calculation costs of cross correlation and the time resolution of the cross-correlation function. However, the calculation costs of cross correlation by single-bit signal processing are constant and independent of the sampling frequency of digital signal processing. Therefore, cross correlation by single-bit signal processing is an effective method for improving the time resolution of the TOF despite the computational ability of autonomous mobile robots.

This paper examines ultrasonic distance measurement with high-time-resolution cross-correlation function obtained by single-bit signal processing. The distance errors of ultrasonic distance measurement are brought about by the proposed signal processing, the noise included in the received signal, and fluctuations in the air. Therefore, the estimated distances were evaluated based on computer simulations and the experimental results.

2. CROSS CORRELATION BY SINGLE-BIT SIGNAL PROCESSING

2.1. Recursive Cross-Correlation Operation

Cross correlation by single-bit signal processing in the proposed method consists of a recursive cross-correlation operation of single-bit signals and a smoothing operation accomplished by the FIR low-pass filter, as illustrated in Fig. 1. In the proposed method, an LFM signal is transmitted by a loudspeaker. The received signal of a microphone is converted into a single-bit received signal $x_1(t)$ by a delta-sigma modulator. The transmitted LFM signal is converted into a single-bit reference signal $h_1(i)$ of $N$ samples by a digital comparator. The cross-correlation function $c_1(t)$ of the received signal $x_1(t)$ and the reference signal $h_1(i)$ is expressed as

$$c_1(t) = \sum_{i=0}^{N-1} h_1(N-i) \cdot x_1(t-i).$$

The calculation of the cross-correlation operation of Eq. (1) requires huge numbers $N$ of multiplications and accumulations of single-bit signals.

The difference in the cross-correlation function, $c_1(t) - c_1(t-1)$, is expressed as

$$c_1(t) - c_1(t-1) = \sum_{i=0}^{N-1} h_1(N-i) \cdot x_1(t-i) - \sum_{i=0}^{N-1} h_1(N-i) \cdot x_1(t-i-1) = h_1(N) \cdot x_1(t) - h_1(1) \cdot x_1(t-N) + \sum_{i=1}^{N-1} (h_1(N-i) - h_1(N-i+1)) \cdot x_1(t-i).$$

The values of $h_1(1)$ and $h_1(N)$ are 1 and $-1$, respectively, because $h_1(i)$ is the LFM signal converted into the single-bit signal. Furthermore, $h_1(i)$ has several hundred zero-cross points $Z_i$. There are the same values, 1 or $-1$, between two zero-cross points $Z_i$ and $Z_{i+1}$ in $h_1(i)$. Therefore, the values of $h_1(N-i) - h_1(N-i+1)$ are expressed as

$$h_1(N-i) - h_1(N-i+1) = \begin{cases} 2, & \cdots, & N-i = Z_{2m-1}. \\ -2, & \cdots, & N-i = Z_{2m}. \\ 0, & \cdots, & N-i \neq Z_i. \end{cases}$$

where $m$ is a natural number. The calculation of the recursive cross-correlation operation, which is performed by integrating the difference in the cross-correlation function, is expressed as

$$c_1(t) = c_1(t-1) - x_1(t-N) + 2 \cdot x_1(t-N+Z_1) - 2 \cdot x_1(t-N+Z_2) + 2 \cdot x_1(t-N+Z_3) - \cdots - x_1(t).$$

The calculation cost of the recursive cross-correlation operation is the integration and summations of single-bit samples. The number of summations $Z_i + 2$ depends only on the number of zero-cross points in the transmitted LFM signal. Therefore, the calculation costs are constant and independent of the sampling frequency of digital signal processing. The recursive cross-correlation operation of single-bit signals thus reduces the calculation costs of cross correlation.

2.2. Smoothing Operation

The single-bit received signal $x_1(t)$ contains a large
The quantity of quantized noise in a high-frequency band because $x_1(t)$ is a delta-sigma modulated single-bit signal. The high-frequency noise decreases the SNR of the cross-correlation function $c_1(t)$. To improve the SNR of $c_1(t)$, the smoothing operation by the moving average filter is required to cancel the high-frequency noise in $c_1(t)$ [10]. The cross-correlation function $c_s(t)$ smoothed by the moving average filter is expressed as

$$c_s(t) = \sum_{i=0}^{M-1} c_1(t - i).$$

(5)

where $M$ is the length of the moving average filter.

The TOF of the received LFM signal is typically estimated from the maximum peak time in $c_s(t)$. The smoothing operation by the moving average filter improves the SNR of the cross-correlation function. However, the smoothing operation by the moving average filter also decreases the maximum peak amplitude in $c_s(t)$. Therefore, the smoothing operation by the triangular weighted moving average filter, which consists of a pair of moving average filters, is proposed. The simulation results of the SNR and the maximum peak amplitude in the cross-correlation function are illustrated in Fig. 2. In the simulation, the frequency of the LFM signal linearly swept from 50 kHz to 30 kHz. The length of the LFM signal was 5 ms. A delta-sigma modulator was the 7th-order delta-sigma modulator. The sampling frequency of the delta-sigma modulator was 12.5 MHz.

The smoothing operation by the triangular weighted moving average filter can increase the SNR and the maximum peak amplitude in the cross-correlation function. In this paper, a 109-taps triangular weighted moving average filter is employed because the SNR and the maximum peak amplitude were more than 40 dB and 0.9, respectively.

3. COMPUTER SIMULATION

3.1. Simulation Parameter

Ultrasonic distance measurement with the high-time-resolution cross-correlation function obtained by single-bit
signal processing was evaluated by computer simulations using MATLAB. The frequency of the transmitted LFM signal linearly swept from 50 kHz to 30 kHz. The length of the LFM signal was 5 ms. The distance to the object was 1 m. The propagation velocity of an ultrasonic wave in air was approximately 345.1 m/s at 22.4°C. The received signal was converted into the single-bit delta-sigma modulated signal by the 7th-order delta-sigma modulator. For comparison, the received signal was also converted into an 8-bit digital signal. The full-scale input range of the 8-bit A/D converter was ±2.5. The sampling frequencies of the single-bit delta-sigma modulated signal and the 8-bit digital signal were 12.5 MHz and 125 kHz, respectively.

The 8-bit received signal was correlated with the 8-bit complex reference signal for interpolation of the cross-correlation function with the phase. The single-bit received signal was correlated with the single-bit reference signal, which was the transmitted LFM signal converted into a single-bit digital signal by the digital comparator. The cross-correlation function of the single-bit received signal and the single-bit reference signal was obtained from the recursive cross-correlation operation of single-bit signals and the smoothing operation accomplished by the triangular weighted moving average filter. The single-bit reference signal had 399 zero-cross points. Therefore, the calculation cost of the recursive cross-correlation operation was the integration and 401 summations of single-bit samples. For the smoothing operation, the length of the triangular weighted moving average filter, which consists of a pair of 55-tap moving average filters, was 109 taps.

3.2. Distance Error by the Proposed Signal Processing

The distance error by the proposed signal processing, cross correlation by single-bit signal processing, was evaluated by computer simulation. The received signal and the cross-correlation function obtained by single-bit signal processing are illustrated in Fig. 3. The received signal includes the transmitted LFM signal and the received LFM signal. The cross-correlation function in Fig. 3 has two peaks, which correspond to the peaks from the transmitted LFM signal and from the received LFM signal, respectively. In the simulation, the TOF of the received LFM signal was estimated from the interval between two peaks in the cross-correlation function. The distance to the object was estimated from the product of the TOF of the received LFM signal and the propagation velocity of an ultrasonic wave in air. The distance to the object was estimated from 500 simulations. The probability distributions of the estimated distance are illustrated in Fig. 4.

The probability distribution of the distance estimated from the real part of the cross-correlation function of 8-bit digital signals is indicated in Fig. 4(a). The distance resolution of the conventional signal processing, cross correlation of 8-bit digital signals, depends on the time resolution of the cross-correlation function. In the case of a
125-kHz sampling frequency, the least significant distance (LSD) is 1.38 mm. The distance resolution of the conventional signal processing can be improved by interpolating the cross-correlation function with the linear approximation of the phase. The phase of the cross-correlation function is estimated from the real and imaginary parts of the complex cross-correlation function with the lookup table of arctangent. The cross-correlation function of 8-bit digital signals is interpolated with the linear approximation of the phase. The probability distribution of the distance estimated from the interpolated cross-correlation function of 8-bit digital signals is indicated in Fig. 4(b). The distance resolution of the cross-correlation function of 8-bit digital signals is improved by interpolation with the linear approximation of the phase. The probability distribution of the distance estimated from the cross-correlation function obtained by single-bit signal processing is indicated in Fig. 4(c). The distance resolution of the proposed signal processing is improved by the high-time-resolution cross-correlation function. In the case of 12.5-MHz sampling frequency, the LSD is 0.0138 mm. The distance error of ±1 sample is brought about by the error in cross correlation by single-bit signal processing. The proposed signal processing can realize high resolution at a distance without interpolation of the cross-correlation function.

3.3. Distance error by the Noise Included in the Received Signal

The distance error by the noise included in the received signal was evaluated by computer simulation. In the simulation, the SNR of the reflected echo was changed by adding normally distributed random noises to the received signal. The received signals and the cross-correlation functions obtained by single-bit signal processing are illustrated in Fig. 5. In the case of each SNR, the distance to the object was estimated from 500 simulations. The probability distributions of the estimated distance are illustrated in Fig. 6. Averages and standard deviations of the estimated distances are indicated in Tables 1, 2.

In the case of the interpolated cross-correlation function of 8-bit digital signals, the standard deviations of the estimated distances increased as the SNR of the reflected echo decreased. In the case of the cross-correlation function obtained by the single-bit signal processing, however, the deterioration of the estimated distance’s standard deviation can be suppressed. Standard deviations of the estimated distances are improved because the noise included in the received signal is reduced by the smoothing operation using the triangular weighted moving average filter. Therefore, the accuracy of distance in a noisy environment is improved by the high-time-resolution cross-correlation function obtained by single-bit signal processing.

4. EXPERIMENT IN ULTRASONIC DISTANCE MEASUREMENT

4.1. Experimental Setup

Cross correlation by single-bit signal processing was evaluated by experiments in ultrasonic distance measurement. The experimental setup for ultrasonic distance measurement is illustrated in Fig. 7. The frequency of the transmitted LFM signal, which is the frequency band suitable for the loudspeaker, linearly swept from 50 kHz to 30 kHz. The length of the LFM signal was 5 ms. The LFM signal was generated from the function generator and amplified by the amplifier. The ultrasonic pulses were transmitted from the loudspeaker, and the echo reflected from the wall was then detected by the microphone. The distance $d_1$ to the wall from the center of the loudspeaker
Fig. 6 The probability distributions of the estimated distance in the simulations. (a), (d), (g): the distance is estimated from the cross-correlation function of 8-bit digital signals, (b), (e), (h): the distance is estimated from the cross-correlation function of 8-bit digital signals interpolated with the linear approximation of the phase, (c), (f), (i): the distance is estimated from the cross-correlation function obtained by single-bit signal processing.

Table 1 Averages of the estimated distances in the simulations.

<table>
<thead>
<tr>
<th>SNR [dB]</th>
<th>Average of distance [m]</th>
<th>Interpolation</th>
<th>Proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$1 + 3.14 \times 10^{-6}$</td>
<td>$1 + 1.97 \times 10^{-6}$</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$1 + 4.73 \times 10^{-6}$</td>
<td>$1 + 0.53 \times 10^{-6}$</td>
<td></td>
</tr>
<tr>
<td>-10</td>
<td>$1 + 39.3 \times 10^{-6}$</td>
<td>$1 + 0.34 \times 10^{-6}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 Standard deviations of the estimated distances in the simulations.

<table>
<thead>
<tr>
<th>SNR [dB]</th>
<th>Standard deviation of distance [μm]</th>
<th>Interpolation</th>
<th>Proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>9.02</td>
<td>9.81</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>27.2</td>
<td>15.6</td>
<td></td>
</tr>
<tr>
<td>-10</td>
<td>662</td>
<td>40.6</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 7 Experimental setup for ultrasonic distance measurement.
and the microphone was 1.000 m. The distance \( d_0 \) between the loudspeaker and the microphone was 0.100 m. The distances were measured by a laser distance meter. The propagation velocity \( v_0 \) of an ultrasonic wave in air was approximately 345.1 m/s at 22.4°C.

The received signal detected by the microphone was converted into the single-bit delta-sigma modulated signal by the 7th-order delta-sigma modulator, and the single-bit received signal was then uploaded to the computer through an oscilloscope. The sampling frequency of the single-bit received signal was 12.5 MHz because the main clock frequency of the 7th-order delta-sigma modulator was 12.5 MHz. For comparison, the received signal was converted into an 8-bit digital signal, and the 8-bit received signal was then also uploaded through the oscilloscope. The oscilloscope’s full-scale input range was \( \pm 0.25 \) V. The sampling frequency of the 8-bit digital signal was 250 kHz.

In the computer using MATLAB, the single-bit received signal was correlated with the single-bit reference signal, which was the transmitted LFM signal converted into the single-bit digital signal by the digital comparator. The cross-correlation function of the single-bit received signal and the single-bit reference signal was obtained from the recursive cross-correlation operation of single-bit signals and the smoothing operation accomplished by the triangular weighted moving average filter. The filter length was 109 taps. For comparison, the 8-bit received signal was correlated with the 8-bit complex reference signal for interpolation of the cross-correlation function with the phase.

### 4.2. Distance error by Fluctuations in the Air

The distance error by fluctuations in the air was evaluated by the experimental results. The received signal and the cross-correlation function obtained by single-bit signal processing are illustrated in Fig. 8. The received signal includes the transmitted LFM signal and the received LFM signal. The cross-correlation function in Fig. 8 has two peaks, which correspond to the peaks from the transmitted and received LFM signals. In the experiments, the TOF of the received LFM signal was estimated from the interval between two peaks in the cross-correlation function. The distance to the wall is estimated as

\[
d = \sqrt{\frac{(TOF \cdot v_0 + d_0)^2 - d_0^2}{2}}.
\]

The distance to the wall is estimated from 100 experiments. The probability distributions of the estimated distance are illustrated in Fig. 9.

In the case of the interpolated cross-correlation function of 8-bit digital signals, the average and standard deviation of the estimated distance were \( 1 - 40.7 \times 10^{-6} \) m and 121 \( \mu \)m, respectively. In the case of the cross-correlation function obtained by the single-bit signal processing, the average and standard deviation of the

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**Fig. 8** The received signal and the cross-correlation function obtained by single-bit signal processing in the experiment.

**Fig. 9** The probability distributions of the estimated distance in the experiments. (a): the distance is estimated from the cross-correlation function of 8-bit digital signals, (b): the distance is estimated from the cross-correlation function of 8-bit digital signals interpolated with the linear approximation of the phase, (c): the distance is estimated from the cross-correlation function obtained by single-bit signal processing.
estimated distance were $1 - 25.5 \times 10^{-6}$ m and 131 $\mu$m, respectively. The distance errors by fluctuations in the air are higher than the errors by the noise included in the received signal.

### 4.3. Distance Error by the Noise Included in the Received Signal and Fluctuations in the Air

The distance error by the noise included in the received signal and fluctuations in the air was evaluated by the experimental results. In the experiments, the SNR of the reflected echo was changed by transmitting normally distributed random noises from another loudspeaker to the received signal. The received signals and the cross-correlation functions obtained by single-bit signal processing are illustrated in Fig. 10. In the case of each SNR, the distance to the object was estimated from 100 experiments.

The probability distributions of the estimated distance are illustrated in Fig. 11. Averages and standard deviations of the estimated distances are indicated in Tables 3, 4.

In the case of the interpolated cross-correlation function of 8-bit digital signals, standard deviations of the estimated distances increased when the SNR of the reflected echo was $-10$ dB. The maximum peak amplitude in the cross-correlation function of 8-bit digital signals is lower than that in the cross-correlation function obtained by single-bit signal processing, as illustrated in Fig. 12. In the case of the cross-correlation function of 8-bit digital signals, the distance could not be occasionally estimated from the maximum peak time in the cross-correlation function when the SNR of the reflected echo was $-10$ dB. Therefore, the accuracy of distance in a noisy environment is improved by the high-time-resolution cross-correlation function obtained by single-bit signal processing.

### 5. CONCLUSION

Distance measurement using cross-correlation by single-bit signal processing is evaluated by comparison with cross-correlation of multi-bit digital signals and interpolation of cross-correlation function with the linear approximation of the phase based on computer simulations and the experimental results. The noise included in the received signal was reduced by the smoothing operation using the triangular weighted moving average filter. In addition, the high-time-resolution cross-correlation function can obtain higher peak amplitude than the cross-correlation function of 8-bit digital signals. Therefore, distance measurement using cross-correlation by single-bit signal processing can improve the accuracy of distance in a noisy environment.
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REFERENCES


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